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FAMILY OF HIERARCHICAL MODELS OF SEQUENTIAL OPTIMIZATION OF THE ENERGY SYSTEMS SUSTAINABLE DEVELOPMENT

Specific properties of the energy systems lead to conclusion, that cannot be created one absolute mathematical model. Needs different complexes and whole families of mathematical models, differentiated by the goals and objectives of management, the type and purpose of the system, the level of detail and the like. The hierarchical mathematical models family, of the successive refinement of decisions, proposed.

Keywords: model; hierarchical structure; sustainable development.

One of the key aspects of system researches in power engineering is mathematical modeling at the optimal control of energy systems. The process of creating the acceptable complexity models, logically leads to the hierarchical principle of their construction, including successive refinement of decisions. One of the objective properties of the energy systems is the heterogeneity of structure, inherent in any complicated systems. The energy systems specific properties define the requirements to mathematical models and methods. Firstly, cannot be created one absolute mathematical model, for example, of energy of country or her main constituents. Needs different complexes and whole families of mathematical models, differentiated by the goals and objectives of management, the type and purpose of the system, the level of detail and the like. The family of hierarchical mathematical models is proposed.

1. The classic model of the controlled dynamical system, is generalized in the hierarchically controlled dynamical system model, which contains $\boldsymbol{r} \in \boldsymbol{R}$ levels administrative-territorial

hierarchy and the industrial infrastructure detailed in the structure of their technological $\boldsymbol{k} \in \boldsymbol{K}$ content.

The task of hierarchical control such a dynamic system is formulated in the following way:

$$\boldsymbol{\Omega_{\tau r k}} \in \boldsymbol{\Phi_{\tau r k}} | \boldsymbol{\tau} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T};$$
$$\boldsymbol{\Omega_{\tau r k}} | \boldsymbol{u}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}), \boldsymbol{\xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}) \Rightarrow \boldsymbol{\Omega_{(\tau+1) r k}} | \boldsymbol{\tau} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T};$$
$$\boldsymbol{\mu_{\tau r k}} = \sum_{r=1}^{R} \sum_{k=1}^{K} \boldsymbol{g}(\boldsymbol{\Omega}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}), \boldsymbol{u}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}), \boldsymbol{\xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k})) \rightarrow \boldsymbol{max};$$
$$\boldsymbol{u}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}) \in \boldsymbol{U}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}) | \boldsymbol{\tau} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T};$$
$$\boldsymbol{\xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}) \in \boldsymbol{\Xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k}) | \boldsymbol{\tau} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T};$$
$$\boldsymbol{\Omega_{\emptyset r k}} \in \boldsymbol{\Omega_{\emptyset}} | \boldsymbol{\tau} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{T};$$

where: $\boldsymbol{\Omega_{\tau r k}}$ – the state vector of structure of the technological content $\boldsymbol{k}$ of the level $\boldsymbol{r}$ at the moment $\boldsymbol{\tau}$;

$\boldsymbol{\Phi_{\tau r k}}$ – the set of feasible states;

$\boldsymbol{u}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k})$ – the control actions vector;

$\boldsymbol{\xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k})$ – vector of random external influences;

$\boldsymbol{\mu_{\tau r k}}$ – the optimality criterion;

$\boldsymbol{U}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k})$, $\boldsymbol{\Xi}(\boldsymbol{\tau}, \boldsymbol{r}, \boldsymbol{k})$ – the set of possible values of controls and random external influences;

$\boldsymbol{\Omega_{\emptyset}}$ – known initial system state; $\boldsymbol{T}$ – the simulation period.

2. Dynamic models based on the principle of input-output, which additionally entered the equation describing linkages change over time based on certain indicators, such as capital investments and fixed assets, providing a balance of continuity between the individual periods [1]. Thus, we get a description of the cycle of reproduction, usually for a period of implementation – from the creation of funds to identify increased as a result of their use of production capacity. The models usually represented as a set of balance of production and capital investments, demand for which is set for future steps by

rationing under construction. The problem of finding the optimal scenario the system [2], considered as a gradual sequence of optimal balances. For Leontief's type model "investments - generation - supply - consumption" takes into account the timing of the introduction of new and upgraded generation systems, dispensing and delivery, including taking into account the necessary supply of material and production resources. The fundamental limitation of the model with the reserves and balanced equilibrium growth were used. In the process of calculating scenario the problem of minimum total cost of generating electric power for the period of simulation was solved:

$$\boldsymbol{Q^{gen}} = \sum_{\boldsymbol{\tau=1}}^{\boldsymbol{T}} \sum_{\boldsymbol{J}} \boldsymbol{Q_{\tau}^{J}} \Rightarrow \boldsymbol{min}, | \, J = NPP, HES, TPP, CHP, REN\,;$$

The baseline, pessimistic and optimistic scenarios of generation and consumption were calculated.

3. The problem of search of optimum algorithm of management of energy system, taking into account the randomness of the external action formulated in terms of stochastic control theory with adaptation [3]. The optimal procedure for sequential process with a finite number of steps and quadratic loss function at every step, determined. Such type of the loss function allows the use of optimal solution procedures based on linear functions of control actions.

Let, $\boldsymbol{\Omega_{\emptyset}}, \dots, \boldsymbol{\Omega_{k}}$ – finite sequence of States of the stochastic hierarchically controlled dynamical system at different steps of a sequential process. $\boldsymbol{\Omega_{\emptyset}}$ – initial state of the system, $\boldsymbol{\Omega_{1}}, \dots, \boldsymbol{\Omega_{\tau}}$ – the states of the system in the following steps. The process ends if at any step $\boldsymbol{\tau}$ value $\boldsymbol{\Omega_{\tau}}$ different from target value $\boldsymbol{t_{\tau}}$ less than or equal to a certain $\boldsymbol{\varepsilon}$.

If at some step τ *(τ = 1, ..., T)* the distribution of the next state $\Omega_{\tau+1}$ depends only on present state Ω_τ and the value of the control u_τ, the process can be described by the following system of equations: $\Omega_{\tau+1} = \alpha_\tau \Omega_\tau + \beta_\tau + u_\tau + \xi_\tau$ (1)

α_τ, β_τ – constants, $\alpha_\tau \neq 0$, u_τ – control value that you select in the next step τ based on the values Ω_τ and ξ_τ, that is, normally distributed random variables with zero mean and variance γ_τ^2. $\xi_1, \ldots, \xi_T$ – random perturbations of the system that are considered independent. $\Omega_\emptyset$ – known initial system state.

For the optimal trajectory system motion need $a_{\tau-1}, b_{\tau-1}$ compute by the following formulas [3]:

$$a_{\tau-1} = \left(\frac{\propto_\tau^2 \rho_\tau(\theta_\tau + a_\tau)}{\theta_\tau + a_\tau + \rho_\tau}\right), \quad b_{\tau-1} = \frac{1}{\propto_\tau}\left(\frac{\theta_\tau t_\tau + a_\tau b_\tau}{\theta_\tau + a_\tau} - \beta_\tau\right)$$

were ρ_τ – losses associated with selection management u_τ, θ_τ – value of the deviation Ω_τ from target value t_τ. u_τ can be calculated as a linear function of Ω_τ when known t_τ and a given value of loss at each step according to the following formula:

$$u_\tau = \frac{\theta_\tau t_\tau + a_\tau b_\tau - (\theta_\tau + a_\tau)(\propto_\tau * \Omega_\tau + \beta_\tau)}{\theta_\tau + a_\tau + \rho_\tau}$$

Found the values of the controls u_τ not depend on the shape and magnitude of the variances of the distributions of random disturbances $\xi_1, \ldots, \xi_T$ provided, that these perturbations are independent with zero mean and finite variance. In addition, the values of the controls remain optimal for any process described by a system of equations (1).

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3. Morris H. DeGroot. Optimal Statistical Decisions. – May 2004, 489 p.